Quark Masses and Renormalization Constants from Quark Propagator and 3-point Functions*

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We have computed the light and strange quark masses and the renormalization constants of the quark bilinear operators, by studying the large-p^2 behaviour of the lattice quark propagator and 3-point functions. The calculation is non-perturbatively improved, at $\mathcal{O}(a)$, in the chiral limit. The method used to compute the quark masses has never been applied so far, and it does not require an explicit determination of the quark mass renormalization constant.

1. INTRODUCTION

In this talk we present the results of two different lattice calculations. The first one is a non-perturbative determination of the light and strange quark masses, based on the study of the large-p^2 behaviour of the renormalized quark propagator. This method has never been applied so far, and it has the advantage of not requiring an explicit evaluation of the quark mass renormalization constant. The second calculation is a non-perturbative determination of the renormalization constants of the bilinear quark operators, performed by using the non-perturbative renormalization method (NPM) [1]. In both calculations we have used a sample of 100 gauge fields configurations generated, in the quenched approximation, on a $24^3 \times 64$ lattice, at $\beta = 6.2$. The quark propagators have been computed by using the non-perturbatively $\mathcal{O}(a)$-improved Alpha action [2], at four different values of the light quark masses.

The NPM, used to compute the lattice renormalization constants, is standard by-now, and it will not be discussed here in further details. The procedure followed to determine the light quark masses is based on an axial-vector Ward identity, at zero momentum transfer, which relates the quark propagator, $S(p)$, to the amputated Green function of the pseudoscalar density, $\Lambda_5(p)$, com-

puted between external (off-shell) quark states of momentum p:

$$2m_q(\mu)\widehat{\Lambda}_5(p;\mu) = \gamma_5\widehat{S}(p;\mu)^{-1} + \widehat{S}(p;\mu)^{-1}\gamma_5 \quad (1)$$

All quantities in eq. (1) are renormalized in the same scheme, and at the same renormalization scale μ. In the RI-MOM scheme, the Green function $\widehat{\Lambda}_5(p;\mu)$ satisfies, in a fixed gauge, the following condition: $(1/12) \, \mathrm{Tr}[\gamma_5\widehat{\Lambda}_5(p;\mu)]_{p^2=\mu^2} = 1$. By tracing both sides of eq. (1) with γ_5 and choosing $p^2 = \mu^2$, one obtains:

$$m_q^{\mathrm{RI}}(\mu) = \frac{1}{12} \, \mathrm{Tr}\left[\widehat{S}(\mu;\mu)^{-1}\right] \quad (2)$$

Thus, the renormalized quark mass can be computed by studying the large-p^2 behaviour of the renormalized quark propagator.

The results presented in this talk aim to be non-perturbatively $\mathcal{O}(a)$-improved. However, we have been able to improve the quark propagator and the 3-point functions only in the chiral limit. The difficulties arise from the fact that these quantities are both off-shell and gauge non-invariant. In this case, in order to achieve the improvement, three distinct classes of higher-dimensional operators have to be included in the mixing with the lower-dimensional operators [3]. These are: i) gauge-invariant operators, which do not vanish by the equation of motion; ii) gauge non-invariant operators, which are, however, allowed by BRST symmetry; iii) operators vanishing by the equation of motion, which can be either

gauge-invariant or non-invariant, and which give only rise to contact terms in the correlation functions. It may be shown that the off-shell $\mathcal{O}(a)$-improvement of the lattice action does not require effectively the inclusion of additional operators, besides the standard Clover term [3]. Thus, in the following, we will only discuss the improvement of the quark and the bilinear quark operators.

2. QUARK MASSES

The improvement of the lattice quark field, q_L, involves the mixing with both a gauge non-invariant operator and an operator vanishing by the equation of motion [3]:

$$\widehat{q} = Z_q^{-1/2} \left[1 + a\, c_q'(\slashed{D} + m_0) + a\, c_{\text{NGI}}\, \overleftarrow{\slashed{\partial}} \right] q_L \quad (3)$$

with $Z_q = Z_q^0 \left(1 + b_q am\right)$. Therefore, in order to improve the quark propagator, three unknown coefficients, namely Z_q, c_q' and c_{NGI}, must be determined. In principle, this could be done by studying the large-p^2 behaviour of the lattice quark propagator. However, in practice, one finds that, up to very small logarithmic and power corrections, only the following combinations of coefficients can be determined at large p^2:

$$(1/12)\,\text{Tr}\left[-i\slashed{p} S_L(p)\right] \simeq Z_q\left[1 - 2a\, c_{\text{NGI}}\, m\right] \quad (4)$$

$$(1/12)\,\text{Tr}\left[S_L(p)\right] \simeq -2a\, c_q' + 2a\, c_{\text{NGI}}\, Z_q \quad (5)$$

where Z_q, in eqs. (4) and (5), is defined in the RI-MOM scheme. Therefore, in order to compute the renormalized, improved propagator, $\widehat{S}(p)$, we have followed an approximate procedure (see ref. [4] for more details). We have subtracted, from the scalar part of the lattice propagator, the constant $\mathcal{O}(a)$-term to which it reduces, asymptotically, at large p^2, cfr. eq. (5). The propagator has been then renormalized by using, as an approximate value of the renormalization constant, the right-hand side of eq. (4). This procedure leaves unsubtracted $\mathcal{O}(amc_{\text{NGI}})$ terms in the renormalized propagator, thus affecting the determination of the quark masses by $\mathcal{O}(g^2am)$ systematic errors. These errors, however, are expected to be negligible in the case of light quarks.

Once the improved renormalized quark propagator has been computed, the procedure to determine the light quark masses, from eq. (2), is straightforward. The renormalized quark mass is extracted, as a function of the bare quark mass, by fitting the trace of $\widehat{S}(p)^{-1}$ at large p^2. This is shown in Figure 1. Then, we extrapolate
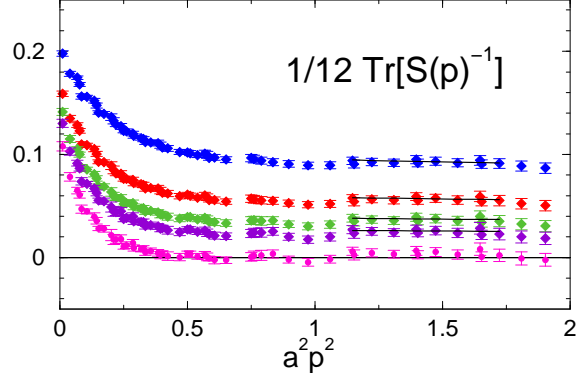

Figure 1. *Trace of the renormalized inverse quark propagator as a function of the scale. The different curves correspond to different values of the bare quark mass, the lowest one being the extrapolation to the chiral limit. The solid lines represent the values of the renormalized quark masses, obtained by fitting the propagator, in the large-p^2 region, as a function of the scale, according to the predictions of the renormalization group equation.*

to the physical values of the light and strange quark masses, by using the experimental values of both the pseudoscalar and the vector meson masses, closely following the procedure described in ref. [5]. For the light and strange quark masses, in the RI-MOM scheme, at 2 GeV, we find:

$$m_l^{\text{RI}} = 5.7(1.0)\,\text{MeV} \quad , \quad m_s^{\text{RI}} = 133(18)\,\text{MeV} \quad (6)$$

These values have been obtained in a complete non-perturbative way. They can be translated into the masses in the $\overline{\text{MS}}$ scheme, by using continuum perturbation theory at the N^{2}LO [6]. At the scale $\mu = 2$ GeV, we find:

$$\overline{m}_l = 4.6(8)\,\text{MeV} \quad , \quad \overline{m}_s = 108(15)\,\text{MeV} \quad (7)$$

3. RENORMALIZATION CONSTANTS

The non-perturbative calculation of the renormalization constants, by using the NPM [1], is becoming, at present, a standard practice in lattice calculations. Therefore, in this section, we limit

Table 1
Values of the lattice renormalization constants obtained by using the NPM and one-loop boosted perturbation theory (BPT). The scheme-dependent renormalization constants are given in the RI-MOM scheme.

	Z_V	Z_A	Z_P/Z_S	$Z_P(2\,\mathrm{GeV})$	$Z_S(2\,\mathrm{GeV})$	$Z_T(2\,\mathrm{GeV})$
NPM	0.757(4)	0.794(6)	0.78(2)	0.44(2)	0.56(2)	0.87(2)
BPT	0.846	0.862	0.954	0.587	0.618	0.942

ourselves to present our results. The only point, which is worth to be mentioned, concerns the off-shell $\mathcal{O}(a)$-improvement of the bilinear quark operators. As discussed in ref. [3], the operator $O_\Gamma = \overline{q}\Gamma q$ also mixes, at $\mathcal{O}(a)$, with an operator which vanishes by the equation of motion:

$$O'_\Gamma = \overline{q}\left[\Gamma(\overrightarrow{\not{p}} + m_0) + (-\overleftarrow{\not{p}} + m_0)\Gamma\right]q \qquad (8)$$

However, this mixing, as well as the $\mathcal{O}(a)$-mixing of the quark field operator, does not contribute to the determination of the renormalization constant in the chiral limit [3,7].

In Table 1 we present our results for the renormalization constant Z_V, Z_A, Z_S, Z_P and Z_T, and compare them with the predictions of one-loop (boosted) perturbation theory. Two remarks are in order at this point. The first one is that previous results for the vector and axial-vector current renormalization constants have been also obtained, for the same action, by studying the lattice chiral Ward identities, in ref. [8]. They found $Z_V = 0.793$ and $Z_A = 0.809$, in good agreement with our determination. The second remark concerns the sizeable discrepancy between the non-perturbative determination of Z_P (but see also Z_S) and the prediction of one-loop perturbation theory. It has been suggested that this discrepancy may be (mostly) due to the presence of a large, non-perturbative, power correction in the correlation function Λ_P, which has not been subtracted in order to compute Z_P [9]. However, we find this not to be the case. To illustrate the point, we show in Figure 2 the behaviour of the (projected) correlation functions Γ_P and Γ_S, and the ratio Γ_P/Γ_S, as a function of the scale. Notice, in particular, that in the region $a^2p^2 \gtrsim 1$, which has been considered to compute the renormalization constants, the ratio Γ_P/Γ_S exhibits a nice plateau, as indeed expected only under the assumption of negligible power corrections.
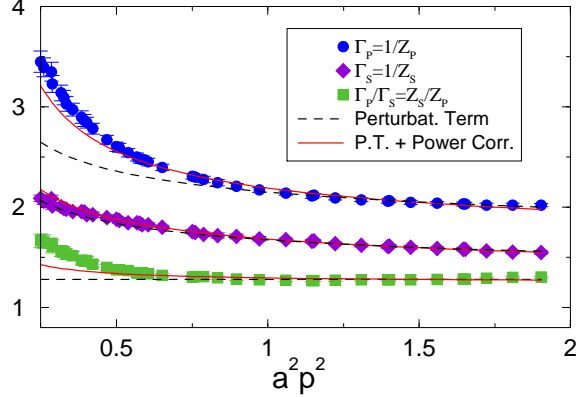

Figure 2. *Green functions of the pseudoscalar and scalar densities, and the ratio Γ_P/Γ_S, as a function of the scale. The dashed lines show the scale dependence predicted by the renormalization group equations at the NLO. The solid lines are the predictions obtained by including the effect of the leading power correction ($\sim 1/p^2$).*

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